Mid-Semestral Exam - Topology I

Max. Marks : 40 Time : $2\frac{1}{2}$ hours

Answer all questions. You may use theorems proved in the class after correctly stating them. Any other claim must be accompanied by a proof.

(1) State whether the following statements are *True* or *False*. Answers without correct and complete justifications will not be awarded any marks.

(a) Let A be the subset of $\mathbb{R}^\omega$ (with the product topology) defined by

$$A = \{x = (x_n) \,:\, \sup_n |x_n| < \infty\}.$$

Then $\mathrm{Int}(A) = \emptyset$.

(b) Let X be an ordered set. If X is connected in the order topology, then X is a linear continuum.

(c) The spaces $\mathbb{Q}, \mathbb{Q}_{>0}$ (with the subspace topology of $\mathbb{R}$) are homeomorphic. $\mathbb{Q}, \mathbb{Q}_{>0}$ denote the rationals and the positive rationals respectively.

(d) If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous and has a local minima at every point, then f is a constant.

(e) Let Y be the collection of all 1-dimensional subspaces of $\mathbb{R}^3$ with the quotient topology determined by the surjective function

$$p : \mathbb{R}^3 - 0 \longrightarrow Y$$

defined by $p(v) = \mathrm{span}(v)$. Then Y is not Hausdorff. [4x5=20]

(2) Let X, Y be topological spaces and $p : X \longrightarrow Y$ a quotient map. If Y is connected and $p^{-1}\{y\}$ is connected for all $y \in Y$, show that X is connected. [6]

(3) Show that every connected open subset of $\mathbb{R}^2$ is path connected. [6]

(4) Consider the cartesian product $\mathbb{R}^J$ where J is an indexing set. Define the uniform metric on $\mathbb{R}^J$ and check that it is indeed a metric. Show that the box topology on $\mathbb{R}^J$ is strictly finer than the uniform topology if J is infinite. Is $\mathbb{R}^J$ connected in the uniform topology? [2+3+3]